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HEAT TRANSFER IN A DENSE MOVING LAYER IN A CYLINDRICAL CHANNEL

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Analytical results are presented on the heat transfer to the layer with rod-type direct motion of the components in a cylindrical pipe; the solutions are analyzed and a comparison is made with experiment.

Our knowledge of heat transfer in moving layers with fluid through flow is inadequate, particularly for the conditions occurring in chemical reactors and other plants. A mathematical description has been given [1], together with the general dimensionless heat-transfer equation and certain approximate relationships. The latter were derived by considering the layer as a pseudocontinuous medium with the components equal in temperature. However, in the general case the temperatures of the gas and solid components are not the same [1-3].

Here we calculate the temperature distribution and heat transfer for a dense layer of this type moving in a cylindrical pipe, with the layer considered as a discrete two-component system, with each of the components acting as a pseudocontinuous medium. The heat transport in each component is represented by the corresponding effective thermal conductivities (λ_g^*, λ_s^*), which incorporate the actual flow structure. A working cell contains a number of particles sufficient for these effective properties to be applicable. The effective thermal conductivity of the solid component incorporates the heat transport by conduction in the particles, as well as by contact between the particles and conduction through the immobile gas near the contacts, in addition to radiative transfer between the surfaces of adjacent particles.

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The effective thermal conductivity of the gas component incorporates the conductive transport, and also radiation and force convection arising from the turbulent mixing. The transport through the solid component occurs in parallel with that through the gas one, while there is convective heat transfer between the two components, which is represented by the intercomponent heat-transfer coefficient α_i. The heat-transfer rate for the flow as a whole against the wall is represented by an overall heat-transfer coefficient, without separate discussion of the core of the flow and the wall region. This approach provides a system of equations that includes the equations of motion, continuity, and energy for the components, as well as the heat-transfer equation for the boundary. The system is too complex to be solved analytically, but the problem can be greatly simplified if it is assumed that the components move in a rod-type fashion, the basis for which has been discussed previously [1]. In that case, the equations of motion and continuity are eliminated, and the mathematical description involves only the equations for the energies of the components and the heat transfer at the boundary:

$$\rho_g c_{pg} v_{g0} (1-\beta) \frac{\partial \vartheta_g}{\partial x} = \lambda_g^{*} \left(\frac{\partial^2 \vartheta_g}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta_g}{\partial r} + \frac{\partial^2 \vartheta_g}{\partial x^2} \right) + a\alpha_i (\vartheta_s - \vartheta_g) \pm q_{vg}, \tag{1}$$

$$\rho_s c_s v_{s0} \beta \frac{\partial \vartheta_s}{\partial x} = \lambda_s^{*} \left(\frac{\partial^2 \vartheta_s}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta_s}{\partial r} + \frac{\partial^2 \vartheta_s}{\partial x^2} \right) - a\alpha_i (\vartheta_s - \vartheta_g) \pm q_{vg}, \tag{2}$$

$$\alpha = -\left[\lambda_g^{*} \left(\frac{\partial \vartheta_g}{\partial r} \right)_{r=r_0} + \lambda_s^{*} \left(\frac{\partial \vartheta_s}{\partial r} \right)_{r=r_0} \right] \bar{\vartheta}_d^{-1}. \tag{3}$$

The solution is considered subject to boundary conditions of the first kind and the following assumptions:

1) the components move in direct-flow fashion;
2) the physical characteristics of the component are constant;
3) the heat transport by thermal conduction along the flow direction has been neglected by comparison with the convective transport, i.e.,

$$\lambda_g^{*} \frac{\partial^2 \vartheta_g}{\partial x^2} \ll \rho_g c_{pg} v_{g0} (1-\beta) \frac{\partial \vartheta_g}{\partial x}; \quad \lambda_s^{*} \frac{\partial^2 \vartheta_s}{\partial x^2} \ll \rho_s c_s v_{s0} \beta \frac{\partial \vartheta_s}{\partial x}$$

(it is found [4] that the longitudinal thermal conduction in an immobile layer has virtually no effect for $Re_e > 50$);

4) there are no internal heat sources or sinks in the flow;
5) the component temperatures are uniformly distributed in the inlet section;
6) the solid component moves as a close-packed layer, but the motion is unhindered (it is found [1] that these assumptions apply for $Fr = v_{s0}^2/gD \leqslant (0.15-0.5)$ and $D/d > 30$);
7) the relative flow does not affect the mode of motion or the layer packing density; and
8) the packing density does not vary over the cross section.

The boundary conditions take the form

$$x \geqslant 0: t_w = \text{const}, \tag{4a}$$

$$x \geqslant 0,\ 0 \leqslant r \leqslant r_0: v_s = v_{s0} = \text{const};\ v_g = v_{g0} = \text{const}, \tag{4b}$$

$$x = 0,\ 0 \leqslant r \leqslant r_0: \vartheta_s = \vartheta_{s0} = \text{const};\ \vartheta_g = \vartheta_{g0} = \text{const}, \tag{4c}$$

$$x > 0,\ r = r_0: \vartheta_s = \vartheta_g = 0, \tag{4d}$$

$$r = 0: \frac{\partial \vartheta_s}{\partial r} = \frac{\partial \vartheta_g}{\partial r} = 0. \tag{4e}$$

Hankel integral transformation with respect to r and Laplace transformation with respect to x have been used in solving (1) and (2) with the boundary conditions of (4a)-(4d), which reduce the system of (1) and (2) to a system of algebraic equations; the following expressions are obtained for the temperatures of the components:

$$\theta_g = \sum_{n=1}^{\infty} \frac{2}{\mu_n J_1(\mu_n)(p_1 - p_2)} J_0\left(\mu_n \frac{r}{r_0}\right) [(p_1 + M_n)\exp(p_1 x) - (p_2 + M_n)\exp(p_2 x)], \tag{5}$$

$$\theta_s = \sum_{n=1}^{\infty} \frac{2B_2}{\mu_n J_1(\mu_n)(p_1 - p_2)} J_0\left(\mu_n \frac{r}{r_0}\right)\left[\frac{p_1 + M_n}{p_1 + D_n}\exp(p_1 x) - \frac{p_2 + M_n}{p_2 + D_n}\exp(p_2 x)\right]. \tag{6}$$

Here

$$k_n = \frac{\mu_n}{r_0};\ M_n = A_2 k_n^2 + \frac{\vartheta_{s0}}{\vartheta_g} B_1 + B_2;\ E_n = A_1 k_n^2 + A_2 k_n^2 + B_1 + B_2;$$

$$D_n = A_2 k_n^2 + B_2;\ N_n = (A_1 k_n^2 + B_1)(A_2 k_n^2 + B_2) - B_1 B_2;$$

$$p_1 = \frac{-E_n + \sqrt{E_n^2 - 4N_n}}{2};\quad p_2 = \frac{-E_n - \sqrt{E_n^2 - 4N_n}}{2};$$

$$A_1 = \frac{\lambda_g^{\star}}{\rho_g c_{pg} v_{g0}(1-\beta)};\ A_2 = \frac{\lambda_s^{\star}}{\rho_s c_{s0} v_{g0}\beta};$$

$$B_1 = \frac{\alpha_i a}{\rho_s c_{pg} v_{s0}(1-\beta)};\ B_2 = \frac{\alpha_i a}{\rho_s c_s v_{s0}\beta}. \tag{7}$$

The summation in (5) and (6) is over all positive roots of the characteristic equation $J_0(\mu_n) = 0$.

The following expressions define the component temperatures as averaged over the cross section:

$$\bar{\theta}_g = \sum_{n=1}^{\infty} \frac{4}{\mu_n^2} \cdot \frac{1}{p_1 - p_2} [(p_1 + M_n)\exp(p_1 x) - (p_2 + M_n)\exp(p_2 x)], \tag{8}$$

$$\bar{\theta}_s = \sum_{n=1}^{\infty} \frac{4}{\mu_n^2} \cdot \frac{B_2}{p_1 - p_2}\left[\frac{p_1 + M_n}{p_1 + D_n}\exp(p_1 x) - \frac{p_2 + M_n}{p_2 + D_n}\exp(p_2 x)\right]. \tag{9}$$

Equation (3) with (5)-(9) gives the following expression for the local heat-transfer coefficient:

$$\alpha = \frac{w_s + w_g}{2r_0}\left\{\sum_{n=1}^{\infty} \frac{\lambda_g^{\star}}{p_1 - p_2}[(p_1 + M_n)\exp(p_1 x) - (p_2 + M_n)\exp(p_2 x)] + \right.$$

$$+ \left.\sum_{n=1}^{\infty} \frac{\lambda_s^{\star} B_2}{p_1 - p_2}\left[\frac{p_1 + M_n}{p_1 + D_n}\exp(p_1 x) - \frac{p_2 + M_n}{p_2 + D_n}\exp(p_2 x)\right]\right\} \times$$

$$\times \left\{\sum_{n=1}^{\infty} \frac{w_g}{\mu_n^2 (p_1 - p_2)}[(p_1 + M_n)\exp(p_1 x) - (p_2 + M_n)\exp(p_2 x)] + \right.$$

$$+ \left.\sum_{n=1}^{\infty} \frac{w_s B_2}{\mu_n^2 (p_1 - p_2)}\left[\frac{p_1 + M_n}{p_1 + D_n}\exp(p_1 x) - \frac{p_2 + M_n}{p_2 + D_n}\exp(p_2 x)\right]\right\}^{-1}. \tag{10}$$

This is referred to the temperature difference between the wall and the mean integral flow temperature at a given section:

$$\bar{\vartheta}_d = \frac{w_g \bar{\vartheta}_g + w_s \bar{\vartheta}_s}{w_g + w_s}. \tag{11}$$

It is clear from (5), (6), and (10) that the component temperatures and heat-transfer rate are dependent on the speeds and physical characteristics of the component, as well as on the intercomponent heat-transfer coefficient, the geometrical characteristics of the pipe, and so on.

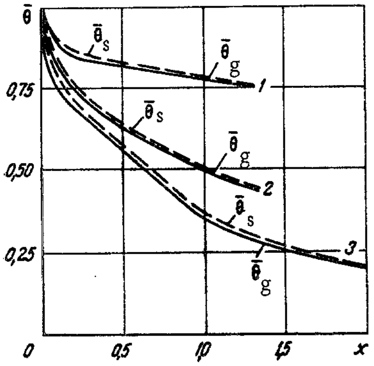

Fig. 1

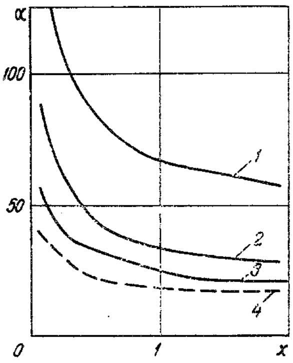

Fig. 2

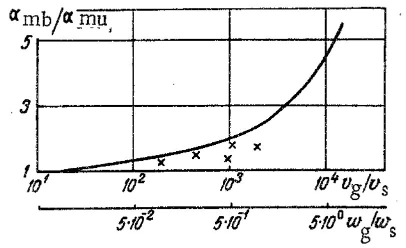

Fig. 3

Fig. 1. Variations along the channel in component temperatures averaged over the cross section: 1) v_{g0} = 1 m/sec; v_{s0} = 0.01 m/sec; 2) v_{g0} = 1 m/sec; v_{s0} = 0.001 m/sec; 3) v_{g0} = 0.1 m/sec; v_{s0} = 0.001 m/sec. x, m.

Fig. 2. Variation in heat-transfer rate along channel: 1-3) bed blown (symbols as in Fig. 1); 4) very low speed ($v_{g0} = v_{s0}$ = 0.001 m/sec).

Fig. 3. Acceleration of heat transfer by blowing in relation to relative speed of components (the points are from experiment [1] for hindered motion in annular channels).

These equations have been used in calculations for an air—ceramic system with the packing moving at speeds between 0.001 and 0.05 m/sec, air speeds from 0.1 to 3 m/sec, pipe diameters from 0.1 to 0.2 m, and particle diameters from 0.003 to 0.01 m. The effective thermal conductivities of the solid and gaseous components were calculated from the formulas of [5], which agree satisfactorily with experiment [6, 7], as does the linear dependence of the effective thermal conductivity of the layer on the Reynolds number. The intercomponent heat-transfer coefficient was calculated from Timofeev's formulas, which apply for a moving layer with a uniform distribution of the components.

We see from (5), (6), (8), and (9) that the variation in component temperature along the pipe is determined by the first term $\exp(p_1x)$ in the brackets, since the value of the second term $\exp(p_2x)$ is negligibly small. Figure 1 shows the temperatures averaged over the cross section as functions of position along the pipe; there is only a slight difference in temperature between the components. Even if the temperatures at the inlet are identical, they become very similar in the initial section ($x \approx 0.1$ m) on account of the intercomponent heat transfer. Under these conditions, the temperature distribution is determined by that of the solid material, whose water equivalent is higher than that of the gas. Figure 2 shows the local heat-transfer coefficient as a function of position for various gas and solid speeds. The thermal stabilization in the initial section causes the heat-transfer coefficient to fall, the value approaching a constant value asymptotically. The heat-transfer rate increases with the speed of either of the components, which agrees with the conclusions of [1]. The length of the stabilization section then increases also. Figure 2 shows also data for a close-packed layer without a difference in flow speed, i.e., the linear speeds of the components are identical, while convective heat transport in the gas component is virtually absent. For identical speeds in the solid, the heat transfer is higher when the gas filters through. The extent of the acceleration in the heat transfer due to this through flow increases with the relative speed of the gas. Figure 3 compares the calculated acceleration factor with the observed values [1]. The qualitative agreement is satisfactory, while the quantitative discrepancies may occur because the experiments were made under conditions different from those assumed in the calculation, particularly with hindered flow of the layer in an annular channel in a countercurrent system.

If one uses modified similarity criteria, which incorporate the characteristics of both components [1], the results from (10) may be approximated almost exactly by the Graetz—Nusselt solution for heat transfer from a homogeneous medium in rod flow, which indicates that the thermal resistance due to the intercomponent heat transfer is negligible in this range of the parameters and has no effect on the heat transfer by the moving mixed layer.

Although various assumptions are involved in deriving this solution, it does provide a quantitative evaluation of the heat-transfer rate for such a layer, as well as the effects of the individual parameters. Subsequently, it will be necessary to consider the process for other boundary conditions, and also to incorporate the actual setting, in particular, the lower effective layer density at the wall and the nonuniform component velocity distributions.

NOTATION

a, surface area of particles in unit volume; c, specific heat; d, particle size; r, current radius; r_0, channel radius; v, velocity; w, water equivalent; β, volume concentration of solid component; λ_g^*, effective thermal conductivity in bed; μ_n, root of characteristic equation; $\vartheta = t - t_w$, local excess temperature; $\theta = \vartheta/\vartheta_{g0}$, local dimensionless excess temperature. Indices: g, gas component; f, flow; w, wall; s, solid component; 0, inlet section (x = 0); mb, moving blown; mu, moving unblown.

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PARTICLE MIXING IN A FLUIDIZED BED CONTAINING BAFFLES

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Results are given on the mixing of the solid phase in a fluidized bed containing baffles; it is shown that a diffusion model can be used to advantage in analyzing longitudinal particle transport.

Mixing in the solid phase in a fluidized bed is a major aspect of fluidization theory, since it provides information on the heat-transfer mechanism and data on the interaction between the gas and solid. Information has been published [1, 2] on the particle mixing in a fluidized bed containing baffles (a structured bed), but the data relate in the main to small laboratory columns and are unrelated to the bed structure. The longitudinal-mixing coefficient correlated with the hydrodynamic factors serving as the basic definitive parameter contains the fluidization number, which unfortunately does not adequately characterize the hydrodynamic setting in a fluidized bed [3, 11]. Here we present results on the solid mixing in larger models of diameters 0.15 and 0.55 m.

The baffles were, as in [10], bent pieces of mesh forming blocks. The specific volume of the baffles was about 2% of the total fluidized-bed volume. The cell size in the mesh was 5 × 5 mm and the wire diameter 1 mm.

Qualitative experiments were made with a planar model with the packing described in [4] to establish the general mode of particle mixing in such beds; stained parti-

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